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From Discrete Intuition to Computational Revolution in the Context of American Chooser Option

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Abstract. This paper illustrates the valuation of chooser options within the broader intellectual lineage of modern option pricing theory, providing both a theoretical and methodological framework. Our analysis is anchored in the discrete-time valuation methodology proposed by Cox, Ross, and Rubinstein, commonly known as the CRR model, which remains one of the most influential and practical approaches for demonstrating no-arbitrage pricing. While acknowledging the continuous-time paradigm of the Black–Scholes–Merton (BSM) model as a theoretical benchmark, we leverage the intuitive and adaptable nature of the binomial framework to deconstruct the chooser option's unique structure. Furthermore, by drawing a conceptual parallel to the work on barrier options by Reiner and Rubinstein, we argue that the analytical treatment of path-dependent but contractually fixed boundaries provides a blueprint for decomposing the chooser's distinctive payoff mechanism. The core contribution of this work lies in the systematic construction of a binomial pricing model tailored to this instrument. We conclude by outlining pathways for future research, including the extension of this framework to the more complex American-style chooser option—a challenge that requires advanced numerical methods such as the Least-Squares Monte Carlo (LSM) algorithm. Finally, this study proposes a testable hypothesis for future validation: that the structural flexibility embedded in the chooser option may justify a higher premium. Further empirical research is needed to confirm this conjecture and to highlight its potential for both practical implementation and continued academic exploration in complex financial contexts.

Keywords: Chooser Option, Binomial Pricing Model, Monte Carlo Simulation

1. Introduction

A chooser option grants its holder the right to decide, at a prespecified date T_1 prior to expiration, whether the contract will become a standard call or put option. According to Hull [1], the value at the decision date equals the maximum of the underlying call and put values. This "mid-life" decision feature makes the instrument structurally distinct from both standard European and American options: it retains the flexibility of a contingent exercise decision while avoiding the complexity of a moving optimal exercise boundary. Although the academic literature on exotic options is extensive—often focusing on derivatives with highly complex structures such as Bermudan, Asian, and compound options—the chooser option has received comparatively limited attention. This is noteworthy because chooser options are not merely theoretical constructs; they have been traded in over-the-counter markets since their introduction in the early 1990s [2].

To develop a flexible and transparent pricing framework, this paper adopts the discrete-time binomial methodology proposed by Cox, Ross, and Rubinstein [3]. Although the continuous-time paradigm of the Black–Scholes–Merton (BSM) model [4] provides a theoretical benchmark and yields closed-form solutions under ideal conditions, its rigid assumptions limit practical adaptability. The CRR model remains the most influential and widely used implementation of risk-neutral valuation, and its backward-induction structure is particularly well suited to instruments with embedded decision points such as the chooser option.

Further methodological insight is drawn from the literature on other path-dependent instruments. For instance, Reiner and Rubinstein [5] defined barrier options as contracts whose payoffs depend not only on the terminal price of the underlying asset but also on the path the asset follows over the option's lifetime. The analogy to the chooser option is clear: just as a barrier's value depends on a contractually fixed trigger condition, the chooser's terminal payoff is determined by a single contractual decision event. This parallel provides a blueprint for decomposing the chooser's payoff structure within a binomial tree framework.

The primary contribution of this paper lies in its methodological approach to pricing the chooser option. Rather than constructing new call or put pricing models, this study systematically adapts and analyzes established frameworks—specifically, the binomial model of Cox, Ross, and Rubinstein [3], the Black–Scholes–Merton model [4], and the Least-Squares Monte Carlo (LSM) method introduced by Longstaff and Schwartz [6]. We begin with concise mathematical formulations to establish the theoretical structure, followed by sensitivity analysis to assess the influence of key parameters on option value. This theoretical foundation is then subjected to rigorous computational verification, including programmatic implementation to compute prices, evaluate numerical stability, and test convergence toward the Black–Scholes benchmark as the number of time steps increases. The paper is organized as follows: Section 2 reviews related literature; Section 3 introduces the core numerical methodologies; Section 4 implements the CRR pricing algorithm with computational examples; Section 5 presents the BSM and LSM implementations; and Section 6 concludes with key findings and directions for future research.

2. Literature Review

The pricing of option contracts is rooted in a rich tradition of option pricing theory, numerical innovation, and computational advances. This section synthesises key results from the literature and places option contracts within this context.

2.1. Foundational Pricing Frameworks

The cornerstone of modern option pricing theory is the continuous-time model proposed by Black and Scholes [4], which provides a closed-form solution for European options under geometric Brownian motion and serves as a theoretical benchmark for valuation. Complementing this, Cox, Ross, and Rubinstein [3] introduced the discrete-time binomial tree model, which simplifies risk-neutral valuation through backward induction in a two-state (up/down) framework. They demonstrated that, as the number of time steps increases, the binomial model converges to the Black–Scholes–Merton (BSM)

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formula, thereby validating its effectiveness as a robust numerical approximation—a critical feature for pricing derivatives such as chooser options that involve embedded decision points.

2.2. Path-Dependent Options and Chooser Structure

Chooser options grant their holders the right to choose between a call or a put option at a fixed decision time T_1, making them a class of path-dependent derivatives. Reiner and Rubinstein [5] laid the groundwork for this field through their analysis of barrier options, whose payoffs depend not only on the terminal price of the underlying asset but also on the specific path that the asset price follows over the life of the contract. Their methodological treatment of fixed contractual trigger conditions provides a conceptual blueprint for understanding and deconstructing the decision-dependent payoff structure of chooser options. Bampou and Dufresne [2] observed that chooser options have been traded over-the-counter since the 1990s, underscoring their practical relevance in financial markets. Martinkute-Kauliene [7] further emphasized their uniqueness: unlike American options with dynamic exercise boundaries, chooser options preserve flexibility through a fixed decision date, combining structural simplicity with contingent optionality.

2.3. Numerical Methods for Complexity

European options can be priced using improved binomial trees or the Black–Scholes–Merton (BSM) framework, but American option variants—which allow early exercise after the decision date—require more advanced numerical methods. The Least-Squares Monte Carlo (LSM) algorithm proposed by Longstaff and Schwartz [6] addresses this issue by estimating the continuation value through regression on simulated paths, thereby enabling the valuation of high-dimensional derivatives. Subsequent studies by Detemple and Emmerling [8] and Qiu and Mitra [9] further demonstrated that this method has become essential for pricing American-style chooser options. Amin and Khanna [10] confirmed that discrete-time models for American-style derivatives, including chooser options, can reliably approximate their continuous-time counterparts, reinforcing the practicality and robustness of both the LSM and binomial-tree frameworks.

2.4. Emerging Frontiers

Nonparametric methods, first introduced by Hutchinson, Lo, and Poggio [11] using neural networks, avoid the strict parametric assumptions of traditional models. Their approach was later extended by Ruf and Wang [12] to exotic derivatives. These studies highlight that using moneyness (e.g., S/K) as an input variable reduces overfitting—a feature particularly critical for chooser options that rely on relative price dynamics. Moreover, the adaptability of neural networks in high-dimensional settings enables the pricing of options with multiple decision points, while careful data partitioning (preserving the time-series structure) helps avoid information leakage. Such models show considerable promise for valuing chooser options under stochastic volatility or jump processes [13]. Sharma and co-authors [14] enhanced the computational efficiency of pricing complex chooser options by introducing GPU acceleration, a development of great importance for real-time risk management. Barria and Hall [15] further validated nonparametric methods using market data, thereby bridging the gap between theoretical modeling and empirical application.

3. Foundational Methodologies in Option Pricing

To properly situate the chooser option pricing model, it is essential to first examine the two cornerstone frameworks of modern option valuation: the discrete-time binomial model and its continuous-time counterpart, the Black–Scholes–Merton (BSM) model, which together form the theoretical foundation for the subsequent analysis.

3.1. Discrete Binomial Model

The binomial model, proposed by Cox, Ross, and Rubinstein [3], demystified option pricing by reformulating it into a simple, discrete-time framework. It assumes that over any small time interval, the asset price can move to one of two possible states—an "up" state or a "down" state. Within this structure, the principles of no-arbitrage and risk-neutral valuation can be applied using straightforward algebra. The model's backward-induction mechanism, starting from the known payoffs at expiration and working backward to the present, makes it an exceptionally powerful and flexible tool. Its structure is particularly well suited to derivatives with early-exercise features or complex, embedded decision points, providing a "simple and efficient numerical procedure" for problems where analytical solutions are unavailable.

3.2. Continuous-Time Benchmark: The BSM Model

The theoretical benchmark for all modern option pricing is the continuous-time model developed by Black and Scholes [4]. Assuming an idealized, frictionless market in which the asset price follows a geometric Brownian motion, the model yields a precise closed-form analytical solution for European options. This elegant formulation provides instantaneous valuation and deep theoretical insights. Cox, Ross, and Rubinstein [3] subsequently demonstrated that their binomial model converges to the Black–Scholes–Merton formula as the number of time steps increases toward infinity. This convergence establishes the BSM model as the theoretical limit of the binomial framework and validates the latter's use as a robust numerical approximation method.

3.3. Numerical Methods Benchmark: The LSM Model

A primary numerical framework for valuing American-style derivatives is the simulation-based model introduced by Longstaff and Schwartz [6]. Their approach is built upon the fundamental concept of continuation value—the expected payoff from holding an option rather than exercising it immediately. Under a set of simulated asset paths, the key innovation of the model is to estimate this continuation value by working backward in time and applying least-squares regression at each decision point. This flexible procedure provides accurate valuations for complex, high-dimensional, and path-dependent American options, for which closed-form solutions are unavailable and traditional grid-based methods become impractical. The model's proven convergence and its slightly downward-biased estimation establish it as a robust numerical technique, validating the use of regression within a Monte Carlo framework to solve the dynamic programming problems inherent in American option pricing.

4. Numerical Calculation of the European Chooser Option

This chapter establishes the mathematical formulation of the chooser option pricing algorithm, adopting the discrete-time binomial methodology of Cox, Ross, and Rubinstein [3] as its starting point.

4.1. Notation & Parameters

The notation used in this study is as follows:

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Table 1. Notation

Notation	Meaning
S_0	Initial asset price (Spot Price)
K	Strike price
r	Annualized risk-free interest rate
σ	Annualized volatility of the asset
T	Maturity of the option (in years)
U	Decision time of the chooser option, with $0 < U < T$
N	Total number of time steps in the binomial tree
$\Delta t = T/N$	Time increment per step

Following the CRR model, the up and down factors per step Δt are given by:

- u: up factor
- d: down factor

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}}. \tag{1}$$

In order to ensure that option prices are determined under no-arbitrage conditions, it is necessary to construct a risk-neutral model, where the risk-neutral probability is given by:

- q: risk-neutral probability

$$q = \frac{e^{r\Delta t} - d}{u - d}. \tag{2}$$

4.2. Binomial Valuation Algorithm

The valuation of a European chooser option is a three-stage backward induction algorithm.

First, the asset price tree is constructed, where the price at any node (t, i):

- t: Step number, with $0 \le t \le N$
- i: Number of up moves, with $0 \le i \le t$

$$S_{t,i} = S_0 u^i d^{t-i}. \tag{3}$$

Second, at the final maturity T, the terminal payoffs for the underlying European call (C) and put (P) are known:

$$C_{T,i} = \max(S_{T,i} - K, 0) \tag{4}$$
$$P_{T,i} = \max(K - S_{T,i}, 0). \tag{5}$$

We then recursively compute the values of C and P at each preceding time step t (for $t = T - 1, \ldots, U$) using the risk-neutral discounting formula:

$$V_{t,i} = e^{-r\Delta t}\left[qV_{t+1,i+1} + (1-q)V_{t+1,i}\right]. \tag{6}$$

This yields the values $C_{U,i}$ and $P_{U,i}$ at each node on the decision date.

Third, at the decision date U, the chooser option's value (V^{ch}) is determined:

$$V^{\text{ch}}_{U,i} = \max\left(C_{U,i}, P_{U,i}\right). \tag{7}$$

With the values at U known, a final backward induction is performed from $t = U - 1$ to $t = 0$ using the same discounting formula to find the final price of the European chooser option, $V^{\text{ch}}_{0,0}$.

$$V^{\text{ch}}_{0,0} = e^{-r\Delta t} \cdot [qV_U^{\text{call}} + (1-q)V_U^{\text{put}}]. \tag{8}$$

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4.3. Numerical Example

In this section, we apply backward induction to price European options across four scenarios that vary in time horizon and price-update frequency. For each scenario, we report the option values, baseline parameters, and key intermediate results.
All other basic parameters are fixed at $S_0 = 100$, $K = 100$, $r = 0.05$, $\sigma = 0.2$. the four scenarios differ only in T, N, and U.

- Scenario 1: Short-Term, Low Frequency (T=1, N=2, U=0.5)
- Scenario 2: Short-Term, High Frequency (T=1, N=4, U=0.5)
- Scenario 3: Long-Term, Low Frequency (T=2, N=2, U=1)
- Scenario 4: Long-Term, High Frequency (T=2, N=4, U=1)

Table 2. Basic parameters

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Δt	0.5	0.25	1	0.5
u	1.152	1.105	1.221	1.152
d	0.868	0.905	0.819	0.868
q	0.554	0.538	0.578	0.554

The following two tables present what we consider the key steps in applying the backward induction method to price a chooser option. These include the terminal values of stock, the option values at the decision time, and the final price $V_{0,0}^{\text{ch}}$.

Table 3. Backward-Induction Results for a European Chooser Option

	Scenario 1	Scenario 2
Terminal call C_T	(0.000, 0.000, 32.690)	(0.000, 0.000, 0.000, 22.140, 49.183)
Terminal put P_T	(24.636, 0.000, 0.000)	(32.968, 18.127, 0.000, 0.000, 0.000)
At U: C_U	(0.000, 17.660)	(0.000, 6.246, 24.609)
At U: P_U	(10.719, 0.000)	(15.658, 3.777, 0.000)
$V_U^{ch} = \max(C_U, P_U)$	(10.719, 17.660)	(15.658, 6.246, 24.609)
$V_{0,0}^{\text{ch}}$	14.204	13.233

Table 4. Backward-Induction Results for a European Chooser Option

	Scenario 3	Scenario 4
Terminal call C_T	(0.000, 0.000, 49.183)	(0.000, 0.000, 0.000, 32.690, 76.065)
Terminal put P_T	(32.968, 0.000, 0.000)	(43.203, 24.636, 0.000, 0.000, 0.000)
At U: C_U	(0.000, 27.017)	(0.000, 9.541, 37.567)
At U: P_U	(13.250, 0.000)	(19.759, 4.663, 0.000)
$V_U^{ch} = \max(C_U, P_U)$	(13.245, 27.017)	(19.759, 9.541, 37.567)
$V_{0,0}^{\text{ch}}$	20.167	19.190

Based on the results from the four scenarios, we reach the following conclusions: holding other parameters constant, an increase in the T enhances the time value of the chooser option, thereby significantly increasing its pricing. When T is fixed, increasing the N will reduce the Δt, that can reduce

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the discretizations error, causing the numerical values to converge toward the limit of continuous time and closer align with the theoretical price. Overall, the option value shows positive sensitivity to T and stable convergence with respect to N. In the next step, we will use program simulation and visualization to more directly illustrate and test the validity of these findings.

5. Programming Implementation and Analysis

To explore the chooser option's properties, we apply the algorithm under two distinct conceptual frameworks, each designed to answer a different set of questions.

5.1. The Sensitivity Analysis Model (Variable T)

To examine the impact of expiration time (T) on the overall option price in the CRR model, we calculated option prices for different values of T, holding a series of fixed parameters, including K, σ, the r, and the $U = 10$, and plotted the trends.

Figure 1 shows that as T increases, both the chooser option price (red line) and the call option price (blue line) show an upward trend. When T is small $(10 < T < 15)$, the price of the chooser option differs significantly from that of the call option. However, as T continues to rise, the two prices gradually converge. This reflects that the flexibility premium provided by chooser options is larger when the expiration time is very short and close to U. However, as the maturity period increases, this flexibility depreciates, causing the chooser option price to approach that of a regular call option.

It is also worth noting that the Put Option price in the chart does not change much, and in fact shows a downward trend. This is because, under the current setting, the price of the underlying asset is high, resulting in a continuous decrease in the probability of a put option being profitable as time goes on, and the increase in its time value is not enough to offset this effect.

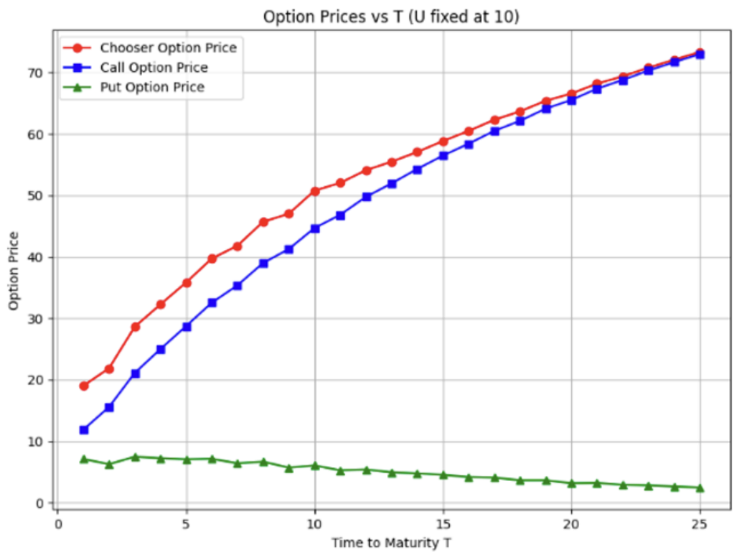

Figure 1: Option Prices vs. Maturity T (with U fixed at 10)

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5.2. Convergence Analysis with Respect to Δt

To assess the numerical stability and convergence rate of the binomial tree implementation for the chooser option, we varied the number of time steps N over a broad range. This experiment isolates the effect of $\Delta t = T/N$ while keeping all other parameters fixed at $S_0 = 100$, $K = 100$, $T = 1.0$, $u_c = 0.5$, $r = 0.05$, and $\sigma = 0.20$. The binomial model prices were compared against the closed-form Black–Scholes value, serving as the ground truth for convergence evaluation.

Figure 1 illustrates the relationship between N and the computed chooser option price. For small N, the pricing error is relatively large due to the coarse time discretization. As N increases, the binomial prices converge monotonically toward the analytical value, with diminishing improvements beyond approximately $N \approx 500$ steps.

To quantify the convergence rate, we computed the absolute pricing error $|V_{\text{binomial}} - V_{\text{BSM}}|$ for each N and plotted the results on a log–log scale (Figure 3). The approximately linear trend confirms the expected $\mathcal{O}(1/N)$ convergence behavior of the standard Cox–Ross–Rubinstein (CRR) scheme. This observation is consistent with the theoretical error bounds for recombining binomial lattices under smooth payoff functions.

While the overall decay trend is evident, the error curve in Figure 3 exhibits small oscillations rather than a perfectly smooth monotonic decrease. This behavior is expected in recombining binomial tree models due to discrete path-dependency and rounding effects in intermediate node valuations. For certain values of N, the discretized tree structure may align more or less favorably with the option's payoff characteristics, leading to minor fluctuations around the asymptotic convergence path. These oscillations tend to diminish in magnitude as N increases, and they do not affect the overall $\mathcal{O}(1/N)$ convergence rate.

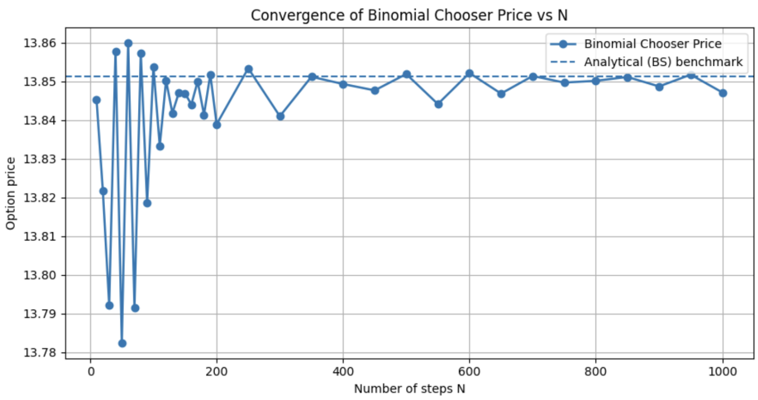

Figure 2: Convergence of binomial tree prices for the chooser option as N increases, compared against the closed-form Black–Scholes price.

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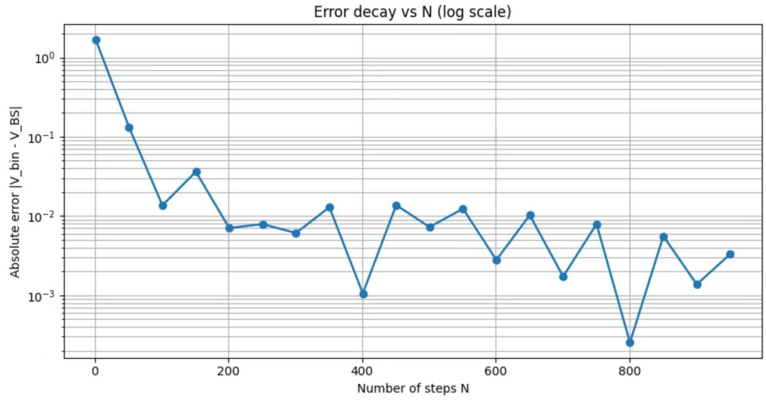

Figure 3: Absolute pricing error decay with respect to N in log–log scale, illustrating $\mathcal{O}(1/N)$ convergence.

6. Theoretical Benchmarks and Computational Frontiers

Having established and implemented the binomial model for the chooser option in the previous chapter, our analysis now extends to two critical paradigms in modern finance. This chapter focuses on the theoretical and methodological application of the continuous-time Black–Scholes–Merton (BSM) model, which serves as the definitive analytical benchmark, and the simulation-based Least-Squares Monte Carlo (LSM) algorithm, which represents the computational frontier for complex American-style chooser options.

6.1. Continuous-Time Benchmark: The BSM Analytical Solution

The Black–Scholes–Merton (BSM) model is arguably the most widely used framework for option valuation, primarily because it provides a closed-form analytical solution for standard European options, as discussed by Martinkute-Kauliene [7]. The model operates under a continuous-time paradigm, assuming that markets are arbitrage-free and that the underlying asset price follows a geometric Brownian motion [1]. For the purposes of this analysis, we consider the simple chooser option, in which the underlying choices are European-style options sharing a common strike price and maturity.

The first phase covers the post-choice period, from the date of decision U to the final maturity T. At the decision point, for any given asset price S_U, the chooser option is equivalent to a portfolio containing a standard European call or a standard European put, and the holder is entitled to select the most valuable at U. The fair market value of each contract is determined directly using the BSM formula, which results in $C(S_U, K, T-U)$ for the call and $P(S_U, K, T-U)$ for the put. A rational holder will choose the one with the highest value, so the value of the chooser option at time U is:

$$V_U = \max\left(C_{\mathrm{BSM}}(S_U, K, T-U),\ P_{\mathrm{BSM}}(S_U, K, T-U)\right) \tag{9}$$

The second phase covers the pre-choice period, from the initial time t_0 to the decision date U. The objective is to determine the present value of the expected future portfolio pay-off. In this setting, the future pay-off is realized at time U and corresponds to V_U. Consequently, the value of the choice option at t_0 is obtained as the discounted expectation of V_U under the risk-neutral measure, we have

$$V_0 = E^Q\left[e^{-rU}\, V_U\right], \tag{10}$$

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hence

$$V_0 = E^Q\left[e^{-rU}\ \max\left(C(S_U, K, T-U),\ P(S_U, K, T-U)\right)\right]. \tag{11}$$

The risk-neutral valuation formula offers an exact theoretical description of the price of the chooser option, but does not produce a closed-form solution for the entire expression in one step. The main difficulty lies in evaluating the expectation operator $E^Q[\cdot]$ over the complex, state-dependent payoff $\max\left(C_{\text{BSM}}, P_{\text{BSM}}\right)$. Therefore, to implement this principle in practice, the Monte Carlo simulation method is employed as a robust numerical method.

6.2. BSM Benchmark via Monte Carlo Simulation

The numerical implementation prices the chooser option via a Monte Carlo simulation. The process begins by generating a large set of asset price paths from the present (t_0) to the decision date (U). At U, for each path, the value of the chooser option is computed with BSM model, with a remaining maturity of $\tau = T - U$. These future values are then discounted to t_0 using the risk-free rate. The resulting average provides a close approximation to the true expected value, with accuracy improving as the number of simulated paths increases.

Under the risk-neutral measure, each terminal price S_U is obtained from the exact solution to the geometric Brownian motion stochastic differential equation:

$$S_U^{(i)} = S_0 \exp\left((r - 0.5\sigma^2)U + \sigma\sqrt{U}\, Z_i\right), \quad Z_i \sim \mathcal{N}(0,1), \quad i = 1, \ldots, n_{\text{paths}} \tag{12}$$

Given the simulated values S_U and the valuation method for the two underlying choices, we proceed as follows. For each simulated price S_U, we determine the chooser option's value at U by taking the maximum:

$$V_U = \max(C_{\text{BSM}}(S_U, \tau),\ P_{\text{BSM}}(S_U, \tau)). \tag{13}$$

These simulated payoffs are discounted to t_0 using e^{-rU}:

$$V_0^i = e^{-rU} \cdot V_U^i. \tag{14}$$

Finally, the present value of the chooser option is obtained by discounting all simulated V_U values to time 0 at rate r and averaging:

$$V_0 \approx \frac{1}{n_{\text{paths}}} \sum_{i=1}^{n_{\text{paths}}} e^{-rU}\, V_{U,i} \tag{15}$$

This averaging process is the numerical counterpart of the expectation operator in the risk-neutral pricing equation. During simulation, a rolling average can be computed to confirm convergence and assess numerical stability.

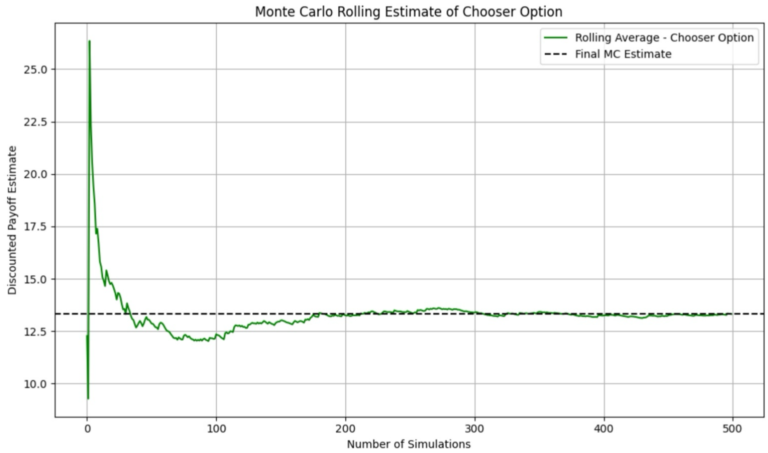

Figure 4: Monte Carlo rolling estimate of the chooser option.
The green line represents the rolling average of discounted payoffs, which gradually converges as the number of simulations increases. The dashed black line indicates the final Monte Carlo estimate.

6.3. LSM Algorithm for the American Chooser

While the BSM framework, in either its closed-form expression or Monte Carlo implementation, offers a rigorous benchmark for the simple (European-style) chooser option, the landscape of option pricing becomes fundamentally more challenging when we introduce American-style features. In the post-choice period, the valuation transforms into a free-boundary problem, where the optimal exercise frontier is not known in advance but must be determined jointly with the option value. Specifically, the payoff structure is governed by the greater of an American call and an American put, each itself the solution to a distinct optimal stopping (obstacle) problem. An effective method to address this complexity is the least squares Monte Carlo (LSM) algorithm introduced by Longstaff and Schwartz [6]. Under a set of simulated asset paths generated in the risk-neutral measure, the LSM approach introduces the concept of a continuation value, the conditional expected cash flow, or payoff, from holding the option rather than exercising it immediately. By working backward in time, the method applies least-squares regression at each decision point to estimate this continuation value, thereby determining the optimal early exercise right.

6.4. Methodological Description of the LSM Model

The Longstaff–Schwartz model offers a flexible and computationally efficient method for valuing path-dependent American-style options, especially in cases where there is no closed-form solution. However, in the context of chooser options, the implementation diverges from the simplified setting of a single American call, requiring a simultaneous evaluation of both embedded call and put positions at the chooser decision date.

The different part is unlike a non-dividend-paying American call should never be exercised early, the chooser framework requires evaluating and comparing the optimal exercise strategies for both an embedded American call and an embedded American put at the chooser decision time U. This entails computing the continuation value for each sub-option independently, then selecting the higher of the two values on a path-by-path basis. Unlike a non-dividend-paying American call, which should never be exercised early, the chooser option involves evaluating and comparing the optimal exercise strategies for both an embedded American call and an embedded American put at the decision time

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U. This requires computing the continuation value of each exercise value independently and selecting the highest of the two values.

Within the LSM algorithm, the cash flow array (`cf`) records, for each simulated path, the cash-flow (`cf`) payoff at its first exercise date that moves forward from maturity. As backward induction proceeds, if an earlier exercise opportunity is found to be optimal, the payoff in `cf.` is replaced with the earlier exercise payoff, and the corresponding exercise time index is updated.

For in-the-money paths at time t, the current underlying price S_t is assigned to the quadratic polynomial basis $\Phi(S_t) = [1,\ S_t,\ S_t^2]$ to form the predictor matrix X. The dependent variable Y is the discounted value of the future cashflow from t. A least-squares regression is then performed to obtain the coefficient vector β, yielding the continuation value equation:

$$\hat{Y} = \beta_0 + \beta_1 S_t + \beta_2 S_t^2. \tag{16}$$

The estimated value calculated $\hat{Y}$ serves as the conditional expectation of the continuous holding value of the chooser option. Compared $\hat{Y}$ with the immediate reward of exercise, it determines whether early exercise is optimal for that path at time t. This backward iteration continues until the initial time is reached.

Finally, these optimal values are discounted to the initial time t_0 and averaged across all simulated paths, yielding the Monte Carlo estimate of the chooser option price, following the same discounting methodology as in the CRR and BSM models.

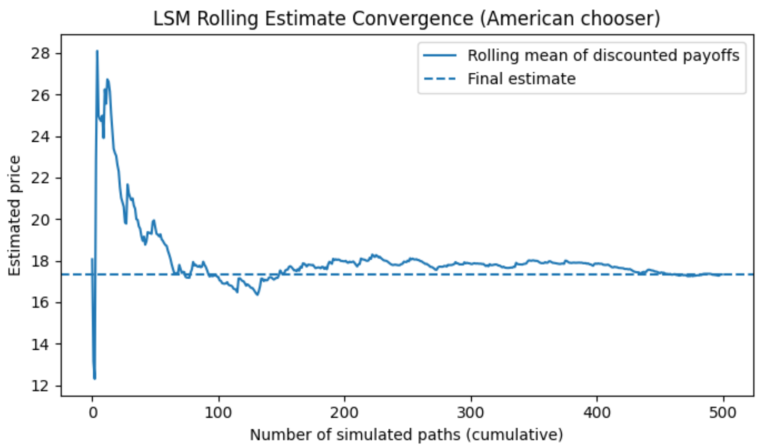

Figure 5: Monte Carlo rolling estimate of the chooser option.
The solid blue line represents the rolling average of discounted payoffs, which gradually converges to a stable value as the number of simulations increases. The dashed blue line indicates the final Monte Carlo estimate.

7. Conclusion and Future Directions

7.1. Conclusion

This paper has systematically constructed and validated a comprehensive pricing framework for chooser options by synthesizing several landmark contributions in the literature. The analysis began with the discrete-time binomial model of Cox, Ross, and Rubinstein [3], which provided a robust theoretical foundation for implementation. It then incorporated the continuous-time paradigm of the Black–Scholes–Merton model [4] as a benchmark, employing the analytical insights of Rubinstein [16] to provide a precise reference. Finally, the theoretical apparatus of the Least-Squares Monte

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Carlo (LSM) algorithm introduced by Longstaff and Schwartz [6] was integrated, offering a modern methodological framework to address the "free-boundary problem" and the "curse of dimensionality" inherent in American-style chooser options.

7.2. Potential Directions for Future Research

To address these challenges, future research could extend the Longstaff–Schwartz (LSM) framework to complex American-style derivatives under more realistic asset-price dynamics, such as stochastic volatility or jump-diffusion processes first examined by Merton [17] and Cox and Ross [13]. The rise of machine learning presents an additional frontier, where neural networks can be trained to learn pricing functions directly from market data without imposing restrictive model assumptions, following the early nonparametric work of Hutchinson, Lo, and Poggio [11]. As discussed by Ruf and Wang [12], the training of an artificial neural network (ANN) involves selecting weight vectors that produce outputs approximating observed option prices, applying a nonlinear activation function ϕ, and optimizing performance using metrics such as mean squared error (MSE). Finally, as models increase in complexity, computational efficiency will become critical; innovations such as GPU-based parallel processing developed by Sharma, Thulasiram, and Thulasiraman [14] will be essential for enabling real-time risk management applications.

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Appendix

The code is available at Chooser Option Pricing Model (GitHub).
Version cited: `v0.1.0`.