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Assessing the Accuracy of Implied Volatility and Historical Volatility in the Black Scholes Merton Model

Peiwei Hong[1,a,*]

[1]*College of Letters and Science, University of California, Santa Barbara, USA*
a. peiwei@ucsb.edu
**corresponding author*

Abstract: This paper demonstrates a comparative analysis of estimating option prices using the Black Scholes Merton model with implied volatility and volatility estimated by historical data. The call option prices were calculated using data from the company Electronic Arts (EA) collected from Yahoo Finance. The analysis suggests that using implied volatility in the Black Scholes model for pricing options yields more accurate estimation than using the volatility calculated from historical data, as attested by comparing the absolute error, error percentage, mean squared error, and financial profit or loss of the two methods. From the results, using implied volatility in the model generate a set of option prices that are closer to the market prices, with an average error percentage of 9.17% and mean squared error of 0.1359, which has significantly lower deviation than using historical volatility in the model, with an average error percentage of 42.46% and mean squared error of 2.4574. A delta-neutral hedging strategy was proposed based on the more accurate method of estimation, utilizing the total delta to reduce the risk of the portfolio. The findings highlight the advantage of implied volatility for option pricing in the Black Scholes model.

Keywords: Option Price Estimation, Black Scholes Model, Historical Volatility, Implied Volatility, Accuracy Assessment.

1. Introduction

There have been numerous financial instruments in the financial markets. Scholars invented various mathematical models to predict the markets of these different financial instruments. Options are one of the types of financial instruments known as a derivative that grants the purchaser the right, but not the obligation, to buy or sell the underlying asset at a fixed price. And the Black Scholes Merton model, invented in 1973 by economists Fischer Black and Myron Scholes, and further expanded by Robert C. Merton, has been one of the most popular mathematical tools to estimate the price of options [1]. Despite the limitations of the Black Scholes model, the model's relative simplicity and accuracy make it a practical tool for the estimation of option prices. In the Black Scholes formula, volatility, denoted by σ (sigma), is a parameter that affects the result of the estimation of option prices.

There are various ways to price options, but the use of Black Scholes model remains dominant among the models. Gianin and Sgarra demonstrated various models to price options, such as using the binomial model and the Black Scholes model. For instance, the binomial model is used for discrete time option pricing, whereas the Black Scholes model can be used for continuous time estimation, and it can be applied to price options with discrete and continuous dividends or without dividends,

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indicating the versatility of the Black Scholes model in its application in various scenarios over other models [2]. In order to follow the procedure, understanding the basics of the Black Scholes model would be primary. Shinde and Takale have provided a thorough explanation of the concepts related to the model, such as the definitions of an option, stochastic differential equation, and Ito's lemma [3].

There are limitations on the assumptions of the traditional framework of the Black Scholes model. For example, the volatility is assumed to be constant in the model [4]. To improve the accuracy of the model, there have been multiple modifications to the parameters and assumptions of the traditional framework. Paula Morales Bañuelos et al. proposed the idea of enhancing the Black Scholes model by using a change of variables [5]. Flint and Maré extended the model by addressing implied volatility calibration in the fractional Black Scholes model, implying that a non-constant implied volatility can yield accurate pricing results [6]. Orzel and Weron discussed the calibration of the Black Scholes models, suggesting that different Black Scholes frameworks may produce better estimations [7]. Their research emphasizes that option pricing model can be improved to overcome the drawbacks of the traditional framework.

The pursuit of more accurate option pricing models has led to a lot of innovative techniques. On another perspective, numerical testing is necessary to examine the feasibility and accuracy of a new modification. Dai et al. modified the Black Scholes model by developing a data driven option pricing methodology that made adjustments to the assumptions and tested pricing options using different ways to obtain the volatility parameter. The results indicate that their method performs very well in general, but is less accurate in some scenarios. To demonstrate the accuracy of different methods, the researcher can consider the proximity between the numerical values of the options using the proposed scheme and the exact values of the options, in which the method is used in Roul's high-order computational approach for the time-fractional Black-Scholes model [8].

As mentioned, the volatility parameter affects the result of the estimation of the option price, so it is beneficial to test the Black Scholes model using different volatility parameters. Brigo explores probability-free models in option pricing, comparing historical and implied volatility, indicating that both measures are viable within the Black-Scholes framework [9].

Constructing a hedging strategy usually follows up after pricing options. Hull and White in "Optimal delta hedging for options" stated that "delta is by far the most important Greek letter [10]." While Alexander and Imeraj emphasize the significance of delta in options, motivating the use of delta, which depend on the volatility [11].

2. Mathematical Formulas

2.1. The Black Scholes Model

The Black Scholes model is a mathematical formula for pricing an option. It is a function of variables:

S – the current option price of the stock.

K – the strike price.

r – the risk-free interest rate.

T – the time in year until the expiration of the option.

σ – the volatility.

N (or Φ) – the standard normal cumulative distribution function.

The formula is defined as:

$$C = \mathrm{N}(d_1)S - \mathrm{N}(d_2)Ke^{-rt} \tag{1}$$

$$P = N(-d_2)Ke^{-rt} - N(-d_1)S \tag{2}$$

Where C = call option price, P = put option price,

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$$d_1 = \frac{\log\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{4}$$

2.2. Implied Volatility

The implied volatility is the volatility that minimizes the sum of squared difference between the market prices of the option and the price estimated by the Black Scholes model. It cannot be directly calculated, but can be obtained by calibration in Excel.

2.3. Historical Volatility

Historical volatility is represented by the annual standard deviation, which depends on the sample standard deviation.

Sample Standard Deviation is defined as:

$$s = \sqrt{\frac{1}{n-1}\Sigma_{i-1}^{n}(x_i - \bar{x})^2} \tag{5}$$

Where x_i = individual data points, $\bar{x}$ = sample mean, n = number of data points.

Historical Volatility/Annual Standard Deviation is defined as:

$$\sigma_{annual} = s \times \sqrt{252} \tag{6}$$

2.4. Sum of Squared Error

The sum of squared error (SSE) is a measurement of the computed values' deviation from the actual values, defined as:

$$SSE = \Sigma_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{7}$$

Where y_i = the actual value of the i-th observation, $\hat{y}_i$ = the predicted value of the i-th observation, and n = the number of observations.

2.5. Delta

Here in the Black Scholes model, the delta (Δ) represents the sensitivity of the option prices to changes in the stock price. The delta for call and put options are calculated using the formulas:

$$\Delta_{call} = N(d_1) \tag{8}$$

$$\Delta_{put} = N(d_1) - 1 \tag{9}$$

Where $N(d_1)$ is the cumulative distribution function (CDF) of the standard normal distribution and

$$d_1 = \frac{\log\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{10}$$

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3. Data and Method

3.1. Data

The data required for the estimation include: stock price, option strike price, maturity (expire date), market price (last buy).

This model assumes the risk-free interest rate r = 0.

All the data can be found on Yahoo Finance.

Company: Electronic Arts (EA)

Stock Price: $148.83 (on Aug 02, 2024)

Table 1 below displays the five chosen call options of the company Electronic Arts. Each of the option price is close to the stock price ($148.83) since options with strike prices closer to the stock price tend to have more consistent prices, thus causing less deviations and improving accuracy in the estimation.

Table 1: Call Options

Options	Expire Date	Strike Price	Last Price
EA240830C00146000	Aug 30, 2024	146	4.2
EA240830C00147000	Aug 30, 2024	147	4.6
EA240830C00148000	Aug 30, 2024	148	3.9
EA240830C00149000	Aug 30, 2024	149	3.1
EA240830C00150000	Aug 30, 2024	150	2.85

Table 2 shows the stock price (close price) of Electronic Arts in a month. The data in Table 2 is primarily used to calculate the annual standard deviation, which is the historical volatility for the model, derived from the sample standard deviation,.

Table 2: Electronic Arts Stock Price by Date

Date	Close
Aug 5, 2024	145.50
Aug 2, 2024	148.83
Aug 1, 2024	148.40
Jul 31, 2024	150.94
Jul 30, 2024	149.12
Jul 29, 2024	147.85
Jul 26, 2024	145.18
Jul 25, 2024	141.80
Jul 24, 2024	141.18
Jul 23, 2024	141.99
Jul 22, 2024	143.25
Jul 19, 2024	140.20
Jul 18, 2024	146.52
Jul 17, 2024	147.00
Jul 16, 2024	146.67
Jul 15, 2024	145.00
Jul 12, 2024	145.68

Table 2: (continued).

Jul 11, 2024	145.30
Jul 10, 2024	144.09
Jul 9, 2024	140.58
Jul 8, 2024	139.65

3.2. Method

3.2.1. Option Price Estimation

The calculations reflect the estimated prices of the call options on Aug 02, 2024, given that they expire on Aug 30, 2024.

The implied volatility in this estimation is the σ (sigma) obtained by using the "What-If Analysis" function in Excel, specifically the Goal Seek tool for calibrating the Black Scholes model.

To estimate the volatility using historical stock data, let S_i be the stock price of the i-th data point, the project can take the natural log of each stock price and calculate the difference between $\mathrm{Ln}(S_{i+1})$ and $\mathrm{Ln}(S_i)$, labeled as Y_i. After calculating the mean of Y_i, squared difference, and sample standard deviation, the project can obtain the annual standard deviation, which is the historical volatility

Using the two volatility parameters, the project can estimate two sets of prices for options EA240830C00146000, EA240830C00147000, EA240830C00148000, EA240830C00149000, and EA240830C00150000 with:

S = 148.83

K = the strike price of each option

T = 25/252 since there are 252 trading days in a year and 25 trading days from Aug 02, 2024 to Aug 30, 2024.

r = 0 under the assumption

σ = the implied volatility and historical volatility

d_1 and d_2 calculated by the formulas

3.2.2. Accuracy Assessment

To assess the accuracy of estimating the option prices using the Black Scholes model with volatility parameters obtained by two methods, the project compared the profit or loss, total absolute error, and mean squared error for each method.

The profit or loss is calculated by assuming one contract per option and each option contract represents 100 shares.

The total absolute error is the sum of the absolute error of each option price estimation.

The mean squared error is calculated from the given formula.

3.2.3. Hedging

The project constructed a hedging strategy based on the more accurate method. By applying delta-neutral hedging, the position in the underlying stock is adjusted to offset changes in the options' values. There are two cases:

If the stock price increases, the call options will gain value, but since the underlying stock is shorted (sold), the profit gained from the options is partially offset by the loss from hedging.

If the stock price decreases, the call options will lose value, but the short stock position will gain value because the stock is sold at a higher price, so the loss from the options is partially offset by the gain from the hedge.

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To construct a hedging strategy, the total delta of the options is utilized to adjust the position to delta-neutral. There are five call options, and the delta for each option is computed in the process of estimating the prices with the Black Scholes model, represented by $N(d_1)$.

4. Results

4.1. Estimated Option Prices and Error Indicators

As shown in Table 3, the estimated price, profit or loss, absolute error, error in percentage, and mean squared error of the estimation, using the implied volatility, are listed. The five estimated option prices are generally close to their market prices, with the smallest difference of $0.0519. The error terms indicate minor to mild deviation in the estimation.

Table 3: Estimation using the Implied Volatility

Calls	Market Price	Estimated Price	Profit/Loss	Absolute Error	Error in Percentage	Squared Difference
EA240830C00146000	4.2	4.5743	-37.43	0.3743	8.91%	0.1401
EA240830C00147000	4.6	4.5481	5.19	0.0519	1.13%	0.0027
EA240830C00148000	3.9	4.4680	-56.8	0.568	14.56%	0.3226
EA240830C00149000	3.1	2.8913	20.87	0.2087	6.73%	0.0436
EA240830C00150000	2.85	2.4370	41.3	0.413	14.49%	0.1706
Total Profit/Loss: -26.87						
Total Absolute Error: 1.6159						
Average Error in Percentage: 9.17%						
Mean Squared Error (MSE): 0.1359						

Table 4 demonstrates the estimated price, profit or loss, absolute error, error in percentage, and mean squared error of the estimation using the historical volatility. The five estimated option prices are relatively not as close to the market prices as the first set of estimation. The error terms confirm the apparent deviation in the estimation.

Table 4: Estimation using the Historical Volatility

Calls	Market Price	Estimated Price	Profit/Loss	Absolute Error	Error in Percentage	Squared difference
EA240830C00146000	4.2	6.3182	-211.82	2.1182	50.43%	4.4868
EA240830C00147000	4.6	5.7571	-115.71	1.1571	25.15%	1.3389
EA240830C00148000	3.9	5.2291	-132.91	1.3291	34.08%	1.7665
EA240830C00149000	3.1	4.7344	-163.44	1.6344	52.72%	2.6713
EA240830C00150000	2.85	4.2726	-142.26	1.4226	49.92%	2.0238
Total Profit/Loss: -766.14						
Total Absolute Error: 7.6614						
Average Error in Percentage: 42.46%						
Mean Squared Error (MSE): 2.4574						

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4.2. Delta and Hedging

As the estimated prices from the estimation with implied volatility are closer to the market price and show less deviations, the delta for each option in the model is presented in Table 5, providing data for a hedging strategy that is more practical to use.

Table 5: Delta for Each Option from Estimation using the Implied Volatility

Calls	Delta
EA240830C00146000	0.65848
EA240830C00147000	0.60720
EA240830C00148000	0.55436
EA240830C00149000	0.50090
EA240830C00150000	0.44778
	Total Delta: 2.7687

5. Conclusion

Volatility is a significant parameter in the Black Scholes model for pricing options, and this paper demonstrates that implied volatility, which is based on current option prices, unlike historical volatility which is based on historical data, is a more accurate parameter for estimating option prices.

This paper has used two methods to estimate five European call options of Electronic Arts with the Black Scholes model. The final results reveal that using implied volatility in the Black Scholes model for pricing options likely yields a more accurate estimation than using historical volatility.

For implied volatility, the estimated prices for the five options are: 4.5743, 4.5481, 4.4680 , 2.8913, and 2.4370 dollars. For historical volatility, the estimated prices for the five options are: 6.3182, 5.7571, 5.2291, 4.7344, and 4.2726 dollars.

Multiple indicators are calculated to assess and compare the two methods of estimating option prices. Assuming one contract per option and each option contract represents 100 shares, the portfolio priced by using implied volatility has a loss of 26.87 dollars. Whereas the portfolio priced by using historical volatility has a loss of 766.14 dollars, which is much higher than the loss of the previous method.

Considering the last buy as the market price for each option, the total absolute error in the case of implied volatility is 1.6159, with an average error percentage of 9.17%. On the other hand, the total absolute error for the estimation with historical volatility is 7.6614, with an average error percentage of 42.46%, suggesting that the estimated prices using implied volatility are closer to the actual market prices of these options.

Besides that, the mean squared error (MSE) for the estimation with implied volatility is 0.1359, and the mean squared error for the estimation with historical volatility is 2.4574. This provides further evidence to the comparison of the two methods.

According to the results, the estimated option prices generated by using the implied volatility have lower loss, total absolute error, error in percentage, and mean squared error than the estimated option prices generated by using the historical volatility. All the indicators imply the higher accuracy of the using the implied volatility in the Black Scholes model for pricing options. In this set of estimation, the delta for each option is: 0.65848, 0.60720, 0.55436, 0.50090, and 0.44778. Hence, the total delta of the five options is 2.7687, indicating that this portfolio is in a net long position. To constructed a hedging strategy based on the more accurate method, this paper proposed selling approximately 2.7687 shares of the underlying stock to adjust to a delta-neutral position. By delta-neutral hedging,

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the portfolio's exposure to changes in the stock price can be reduced, thus reducing the risk of unfavorable price changes.

The results of this project have significant implications for investors and traders in the finance industry who adopt the Black Scholes model to price options. The use of implied volatility likely provides outcomes with higher accuracy, which then can be utilized to construct better hedging strategies and thus reduce financial risk.

Besides testing the Black Scholes model for pricing options using different volatility parameters, one of the key limitations of this study is that it assumes the risk-free rate to be zero, which may not always reflect real market situations. In addition, the estimated values are calculated based on the data of a single company (Electronic Arts). Therefore, further research could broaden this analysis to include multiple assets across various industries to assess whether the conclusions hold.

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