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Comparison of Markowitz Model and Index Model in Different Constraints

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Abstract: This paper reviews the application of Markowitz model and index model in portfolio optimization and focuses on comparing the performance under different constraints. The study covers the core concepts and theoretical framework of modern portfolio theory and the development of the two models. The Markowitz model employs quantitative analysis to consider the expected return and risk of assets and aims to maximize investor utility. The index model, on the other hand, achieves investment objectives by replicating the performance of a specific index. Comparative analysis shows that the Markowitz model is suitable for active management, while the index model is more suitable for passive investment. With the advancement of computing technology, the application of Markowitz model will be more accurate and extensive. Meanwhile, as the concept of passive investment becomes popular, the index model will continue to attract more investors. This study is of great significance to investors in choosing investment strategies, and will help academia and the industry to further optimize investment models.

Keywords: Modern Portfolio Theory, Markowitz Model, Index Model

1. Introduction

The Markowitz model was proposed by Harry Markowitz in 1952. It is a portfolio optimization model aimed at maximizing investor utility by considering both expected returns and risks in the portfolio. The core idea of this model is to reduce risk by diversifying investments while maintaining the expected level of return. It introduces the concept of Modern Portfolio Theory (MPT), which includes the expected value, variance, and covariance of asset returns.[1] The index model is an investment portfolio constructed based on the proportion of constituent stocks in a specific index, such as the S&P 500 index. The goal of this model is to replicate the performance of the index, rather than surpass it. Index models typically have lower management costs and tax efficiency because they do not involve frequent buying and selling operations.

The proposal of the Markowitz model marks a shift in portfolio theory from intuition and empirical rules to quantitative analysis. Over time, this model has been widely used and developed in the financial field, and has become an important tool for investment decision-making. With the development of computer technology, complex mathematical models and algorithms have made the application of Markowitz models more feasible and accurate. The development of index models is closely related to the rise of passive investment strategies. In 1976, John Bogle founded Vanguard

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Group and launched the first index fund - Vanguard 500 Index Fund. The launch of this fund marks the recognition of index investment as a low-cost and efficient investment strategy in the market. With the development of financial markets, the application scope of index models continues to expand, including various types of index funds and exchange traded funds (ETFs).

In practical applications, investors face various constraints, such as liquidity restrictions, transaction costs, tax considerations, legal and regulatory limitations, etc. These constraints may have a significant impact on the selection and optimization of investment portfolios. Therefore, comparing the performance of Markowitz model and exponential model under different constraint conditions is of great significance for understanding their applicability and limitations. Through comparative analysis, investors can better understand which model can better meet their investment goals and needs under specific market environments and constraints. In addition, this comparison also helps the academic and industry further improve and optimize investment models to adapt to constantly changing market conditions.

2. The Historical Development of Markowitz Models

In the initial stage of the Markowitz model, the academic community conducted in-depth research on this theory and consolidated its mathematical foundation. During this period, research on securities return modeling and multivariate normality was its core component. William Sharpe is one of the founders of the Capital Asset Pricing Model (CAPM). The CAPM model was independently proposed by Sharp, John Lintner, and Jack Treynor almost simultaneously in 1964. This model aims to measure the risk and expected return of securities investment and is a core part of modern financial theory. The core idea of the CAPM model is that the expected return on an asset is composed of the risk-free interest rate and the risk premium of the asset relative to the entire market.[2][3] Risk premium reflects the additional return required by investors to bear risks beyond risk-free assets. Sharp's research indicates that the expected return on assets is closely related to their systemic risk, but not to the specific risk of individual assets. Systemic risk refers to the risk factors that affect the entire market, while non systemic risk refers to risks specific to individual assets. In 1965, Eugene Fama proposed the efficient market hypothesis. The efficient market hypothesis suggests that securities prices always reflect all available information, including public and insider information.[4] This means that investors cannot obtain excess profits by analyzing historical data or utilizing any information, as market prices are already the most accurate and comprehensive reflection of information.

In the 1970s and 1980s, with the advancement of computer technology, the Markowitz model was widely used and further developed. The research focus during this period includes expanding the model and increasing empirical research to adapt to the actual market environment and test the applicability of the model. In related studies, Pastor and Stambaugh proposed a model that examined the relationship between market liquidity and expected returns. They found that assets with poor liquidity require higher returns because the cost of buying and selling these assets is higher and it may take longer to complete the transaction.[5] Their research emphasizes the importance of liquidity in asset pricing and the liquidity risks that investors need to assess when considering investment decisions. Michaud conducted an in-depth analysis of the performance of the Markowitz model on actual market data in his research. He found that although the Markowitz model has good theoretical advantages, in actual markets, the practical application effect of the Markowitz model may be affected due to factors such as market friction, transaction costs, and taxation.[6] In addition, Michaud also pointed out that the Markowitz model has a high computational complexity when dealing with large-scale investment portfolios, which may limit its practical application.

In the 1990s and 2000s, with the rapid development of computer technology, numerical methods and software for solving Markowitz model optimization problems made significant progress. These

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improvements make the model more practical and effective in dealing with large-scale investment portfolios. Since the early 21st century, the development of financial theory and practice has prompted researchers to pay attention to multi period and dynamic portfolio selection issues. These studies attempt to combine the Markowitz model with macroeconomic variables, market uncertainty, and investor dynamic preferences to provide a more comprehensive investment decision-making framework.

3. The Historical Development of Index Models

The development of exponential models has gone through three stages. The initial stage can be traced back to the 1970s. At that time, American financial scientist John Bogle founded Vanguard Group and launched the first index fund - Vanguard 500 Index Fund. The launch of this fund marks the recognition of index investment as a low-cost and efficient investment strategy in the market. In the following decades, index models began to expand to other stock market indices, and in this regard, two representative research examples can be mentioned. Firstly, the study of the double index jump diffusion model in the Chinese stock and index markets. This study applies the double index jump diffusion model to the stock and index markets in China. This model was proposed by Wei Xiaodong to address the shortcomings of the Black Scholes model in describing the sharp and thick tailed distribution of asset returns and the "volatility smile" feature. In the application of the Chinese market, research has found that the double index jump diffusion model is superior to traditional jump diffusion models in describing the characteristics of the Chinese stock market.[7] Secondly, stock index prediction modeling based on CEEMDAN-LSTM. This study combines Adaptive Noise Complete Set Empirical Mode Decomposition (CEEMDAN) and Long Short Term Memory Network (LSTM) to construct a stock index prediction model. This method utilizes CEEMDAN to decompose and reconstruct the stock index, and then uses LSTM to predict and model each component. Comparative experiments with existing mainstream prediction methods show that this method performs better in terms of prediction error and lag, and can effectively improve the accuracy of timing strategies.[8]

Since the 2000s, it has been the third stage of the development of index models. With the popularization of passive investment concepts and the continuous expansion of index fund scale, academic and financial industry interest in index models is also increasing. Research has begun to focus on the efficiency, cost-effectiveness, and performance of index models in different market environments. Bilinski proposed an alternative model to CAPM in his 2014 study, which is called "Beta is Shaped". The core idea of this model is that the beta in traditional CAPM models may not be stable, but may be influenced by market participant sentiment and other factors.[9] In the CAPM model, beta is an indicator that measures the systemic risk of assets, reflecting the sensitivity between asset returns and market returns. However, Abdoh and Varela believe that this indicator may be influenced by the sentiment of market participants, such as investors' optimism or pessimism, which may affect their perception of asset risk,[10] thereby affecting asset prices and returns.

In Posch’s study, he delved into the performance of the CAPM model under different market conditions. He chose emerging markets as the research object and used data from emerging markets to test the effectiveness of CAPM.[11] His research found that CAPM has significant limitations in explaining stock returns in emerging markets, indicating that the CAPM model may not be applicable to all market environments. PAOLA's research emphasizes the importance of market specific factors in asset pricing. He believes that each market has its unique characteristics and influencing factors, which may have an impact on asset pricing.[12] Therefore, the conclusion of one market cannot be simply extended to other markets, and separate research and analysis are needed for each market.

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4. An Analysis of the Current Situation of Markowitz Model and Index Model

4.1. Application of Markowitz Model in Portfolio Optimization and Current State of Research

Since Harry Markowitz proposed mean variance analysis in 1952, the Markowitz model has become the cornerstone of modern portfolio theory. This model seeks the optimal asset portfolio under a given expected return level by minimizing portfolio variance. Professor Yu conducted research on the application of the Markowitz portfolio model in the Chinese securities market. He pointed out that due to some strict assumptions of the Markowitz model, there are limitations in its practical application in the Chinese market.[13] To this end, he considered the specific situation of the Chinese securities market and made revisions to certain assumptions in the model to improve its applicability and effectiveness in the Chinese market. Currently, newer research involves improvements to the Markowitz model, application of new technologies or methods, and consideration of more complex market environments. Liu Junfeng use machine learning techniques to optimize investment portfolios. He proposed a machine learning based algorithm that can learn complex relationships between assets from historical data and construct an optimized investment portfolio.[14] This data-driven approach allows researchers to discover patterns that traditional statistical methods may not be able to capture, thereby more effectively achieving risk minimization and profit maximization. And Flood explored the combination of active investment management and passive investment strategies to achieve better risk adjusted returns. He pointed out that although index funds and passive investment strategies have become increasingly popular in the past few decades,[15] in some cases, active management can provide additional value to investors.

4.2. The Application and Research of Index Models in Passive Investment

The application and research of index models in passive investment have always received continuous attention from the academic community. Professor Chen pointed out that with the rise of index investment, the cost of asset management industry is facing pressure, and changes in technology and economies of scale have promoted the commodification of beta products.[16] This means that investors can obtain investment returns that match market indices at a lower cost. In the research of index investment, the evaluation of systematic risk is crucial. Professor Li proposed a model for measuring the risk of individual securities systems, where the systematic risk of securities is determined by their beta coefficients.[17] This study helps investors understand and quantify the systemic risk of investment portfolios. From the development experience of mature markets, passive investment will be a vigorously developing area in the future, especially the broad base index still receiving market attention. Shang Songhui research focuses on the core broad-based indices of the Chinese A-share market, aiming to evaluate the effectiveness and attractiveness of these indices as passive investment tools. In this study, he used a series of quantitative analysis methods to conduct in-depth analysis of the historical performance of the core broad-based index of Chinese A-shares.[18] Zhu Zihao and Qu Hui pointed out in their research that broad-based indices can smooth individual stock risks, track overall market performance, and obtain average market returns. Especially the Science and Technology Innovation 50 and the Shanghai and Shenzhen 300 Index,[19] these two broad base indices have attracted a large amount of net capital inflows, and the product tracking scale has significantly increased.

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4.3. Comparative Analysis of the Advantages and Disadvantages of Markowitz and Index Models

The Markowitz model and the index model are two very different investment strategies, each with its own advantages and disadvantages.The Markowitz model, as the foundation of modern portfolio theory, has the advantage of helping investors maximize their returns on the premise of controllable risk, allowing portfolios to be constructed according to individual risk preferences and return objectives, and has had a profound impact on subsequent investment theory and practice. However, the drawback of the Markowitz model is its computational complexity, especially when dealing with large-scale portfolios, which requires sophisticated mathematical and computational techniques. In addition, the model relies on a variety of assumptions, such as market efficiency and normal distribution of asset returns, which may not always hold true in reality, and the optimization of the model is based on historical data, which may not be able to fully predict future market changes.

In contrast, index models offer the advantages of low cost, simplicity and transparency, and market representativeness. Since passive investment strategies do not attempt to outperform the market, opportunities for excess returns through active management may be missed. In addition, the performance of index funds is closely related to market volatility and therefore may be subject to greater risk in times of market downturns. Overall, the Markowitz model and the index model have their own merits and investors should choose the strategy that best suits their investment objectives, risk appetite, cost considerations and time commitment.

4.4. Comparison of the Scope of Application of Markowitz and Index Models

The scope of the Markowitz Model and Index Model varies depending on the specific needs of the investor and the market environment.The Markowitz Model, the foundation of modern portfolio theory, is suitable for actively managed investors, particularly those seeking to generate above average market returns through active stock selection and market timing. It is also suitable for investors who wish to generate stable returns while taking on a certain amount of risk, as well as investors with specific risk preferences and return objectives that can be customized to fit individual portfolios. Due to the complexity of the model and the need for specialized mathematical and financial knowledge, it is more suitable for professional investors with relevant backgrounds.

While index modeling is more widely applicable. It is suitable for cost-sensitive investors, especially those who are more sensitive to the cost of investment and wish to obtain average market returns through low-cost investment. The index model is also suitable for long-term investors, as index funds do not seek to outperform the market in terms of alpha returns and are more suitable for investors with a long-term investment perspective. It is also suitable for investors who do not wish to trade frequently or conduct in-depth market research, as well as those who wish to achieve broad market diversification by investing in index funds.

5. Prediction of the Development Prospects of Markowitz Model and Index Model

The Markowitz model will continue to benefit from advances in computing technology, especially the application of artificial intelligence and machine learning in finance, which will make the model more accurate and practical. Meanwhile, given the complexity of the real market environment, more market factors are being incorporated into Markowitz models in the future, such as liquidity, transaction costs, and taxes. In addition, the research on market inefficiency in behavioral finance may provide new perspectives for Markowitz model to better explain and predict market phenomena. On the other hand, index models may continue to attract more investors, especially cost-sensitive long-term investors, as the concept of passive investing becomes more popular. It is expected that more index funds and ETFs will be introduced, covering a wider range of asset classes and market

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indices. Although index modeling primarily employs passive management strategies, more products combining elements of active and passive management may be available in the future to provide more flexible investment options.

6. Conclusion

This paper comprehensively analyzes and compares the application of the Markowitz model and the index model to investment portfolios. The Markowitz model, the cornerstone of modern portfolio theory, aims to maximize investor utility by taking into account the expected return and risk of a portfolio of assets and requires specialized knowledge and computational skills. Index models, on the other hand, are portfolios constructed based on the proportion of constituent stocks in a particular index, aiming to replicate the performance of that index, and are suitable for cost-sensitive long-term investors. Both models have their own advantages and applicability scenarios, and investors should choose the appropriate investment strategy based on their own needs and risk appetite in order to achieve their investment objectives. With the advancement of computing technology, the Markowitz model will be more accurate and practical in the future, while the index model will continue to attract more investors with passive investment philosophy. Therefore, investors should choose appropriate investment strategies according to their own needs in order to achieve their investment objectives.

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