

Study on Smoke Screen Interference Shell Deployment Strategy under Different Circumstances

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Abstract. This paper establishes a mathematical model based on the initial position and trajectory of enemy missiles, the positions of our real and decoy targets, the initial position of drones, the number of smoke grenades carried, deployment and detonation timing, and drone allocation. The model aims to find a global optimal solution to enhance the effective coverage duration of smoke clouds and improve drone combat effectiveness. For Problem 1, we first establish mathematical models for drone deployment of smoke interference grenades for enemy missiles M1, drones MY1, real targets, and decoy targets. Subsequently, we calculate their positions at any moment using relevant motion models. Finally, through a line-of-sight (LOS) occlusion model, real targets are abstracted into 16 test points to determine whether missile line-of-sight can be effectively occluded at any moment. The effective coverage duration is calculated by subtracting the initial and final times of effective occlusion. For Problem 2, we first define approximate ranges for four parameters: FY1's flight direction, speed, smoke interference grenade deployment point, and detonation point. We then narrow these parameter ranges using binary search and iteratively solve the objective function for these four parameters using an Evolutionary Search Genetic Algorithm (ESGA) optimized with an early termination mechanism. Genetic algorithm operations identify the maximum effective occlusion duration, with fitness analysis and patience value comparison used to determine whether early termination is necessary to avoid local optima. Finally, sensitivity analysis is conducted using differential evolution and particle swarm optimization models to verify solution accuracy after accounting for uncertain factors.

Keywords: visual line occlusion model, genetic algorithm optimized with early stopping mechanism, differential evolution algorithm, particle swarm optimization algorithm, multi-objective optimization method, NSGA-II, greedy algorithm

1. Introduction

To feed the subsequent model, the section extracts the critical 3-D trajectories of five drones (FY1–FY5) and three incoming 300 m/s missiles (M1–M3) detected at the listed positions, together with the 7 m * 10 m cylindrical decoy (center 0, 200, 0) and the smoke-screen effective range/time data; it then recaps the physics of drone-launched smoke and sets the stage for optimizing release point, burst point and UAV heading/speed (70–140 m/s) in real time.

2. Problem analysis

According to the elements given in the problem, this paper will establish a mathematical model of Cartesian coordinate system containing enemy missiles, our real targets, our decoys, our drones and smoke bombs, so as to facilitate further analysis of the problem.

2.1. Analysis of question 1

Given the known flight speeds and directions of UAV FY1 and missile M1, along with specific timing parameters for smoke screen deployment and detonation, this study first substitutes the geometric center of a real target into the Line-of-Sight (LOS) model [1]. This approach determines the approximate effective occlusion period, followed by precise calculation of the actual effective occlusion duration using 16 feature points.

2.2. Analysis of question 2

Building upon the first research question, this paper employs a ballistic motion model to simulate smoke screen interference munitions' trajectory and utilizes line-of-sight (LOS) occlusion models to evaluate shielding effectiveness. For variables including FY-1's flight direction, velocity, smoke screen deployment coordinates, and detonation points, we apply genetic algorithms. By combining four key parameters as genetic components through hybridization and random mutation processes, we ultimately obtain optimal solutions [2].

3. Assumptions and justifications

Throughout the maneuver, the enemy missile consistently targeted decoys without detecting the actual target. Air resistance was neglected, and the Earth's curvature was disregarded. Assuming the Earth's gravitational acceleration at the incident location is 9.80 m/s^2 , and considering the smoke screen projectile moves in a vertical parabolic trajectory with the drone's velocity after launch, while maintaining a constant descent speed of 3 m/s , the analysis proceeds accordingly [3].

4. Problem 1. establishment and solution of the model

M1 initially heads for the decoy; as its 300 m/s motion shifts the LOS, the screened volume moves accordingly and is tracked by coupling the UAV-smoke shell, ballistic and LOS-geometry models.

4.1. Model building

With the decoy at the origin, the real-target base centre is $(0, 200, 0)$, M_1 at $(20\ 000, 0, 2\ 000)$ and FY_1 at $(17\ 800, 0, 1\ 800)$; the missile trajectory is $x(t)=20\ 000-300\ t$, $y(t)=0$, $z(t)=2\ 000-0.05\ t$:

$$M_1(x, y, z, t) = M_1(20000, 0, 2000, 0) + v_{M1}(x, y, z, t)$$

M_1 's constant 300 m s^{-1} velocity fixes its trajectory: $v_{M1} = (-300, 0, -0.05) \text{ m s}^{-1}$ throughout.:

$$VM1 \begin{cases} \sqrt{x^2 + y^2 + z^2} & = 300 \\ \frac{x}{20000} = \frac{y}{20000} & = \frac{z}{20000} \\ & < 0 \end{cases}$$

The motion trajectory equation of the UAV flight is established by the same idea as follows:

$$F_1(x, y, z, t) = F_1(17800, 0, 1800, 0) + v_{F1}(x, y, z, t)$$

Since the speed and direction of the UAV remain constant after the command is issued, with a speed of 120m/s, the speed of the UAV is also constrained by a set of equations:

$$v_{F1} \begin{cases} \sqrt{x^2 + y^2 + z^2} & = 120 \\ \frac{x}{17800} & = \frac{y}{0} \\ z & = 0 \end{cases}$$

When t is 1.5s, the smoke screen jammer is released and moves with the initial velocity of the UAV at 1.5s:

$$P_1(x, y, z, t) = F_1(x, y, z, 1.5) + v_P(x, y, z, t)$$

Once released, the smoke shell follows a parabola whose initial velocity equals the drone's at separation, so its pre-detonation speed is $v(t) = v_{F1} - gt$:

$$v_P(x, y, z, t) \times z = g \times t$$

After detonation the smoke cloud drifts downward at 3 m s^{-1} , so its vertical coordinate becomes $z(t) = z_{burst} - 3(t - t_{burst})$:

$$B(x, y, z, 0) = P_1(x, y, z, 1.5 + 3.6)$$

$$B(x, y, z, t - 3.6 - 1.5) = B(x, y, z, 0) + v(0, 0, -3)(t - 3.6 - 1.5)$$

Target = 16 rim points; smoke = sphere radius R drifting at 3 m s^{-1} ; LOS M1Qi is broken while $|Q_i(t) - C(t)| \leq R$, and the total span of this inequality is the effective screening time:

$$L(t) = M_1 + t(Q_i - M_1), t \in [0, 1], i \in \{i \in \mathbb{Z} | 1 \leq i \leq 16\}$$

When a point on the line is a distance of r from the center of the sphere C , the point lies on the sphere:

$$|IL(t) - C| = r$$

$$\| P + t(Q_i - M_1) - C \|^2 = r^2, i \in \{i \in \mathbb{Z} | 1 \leq i \leq 16\}$$

Let the line segment direction vector be i , The vector of the missile to the center of the ball is :

$$\vec{d}_i = \vec{Q_iP}$$

$$l = \|\vec{d}_i\|$$

$$\hat{\vec{d}}_i = \frac{\vec{d}_i}{\|\vec{d}_i\|}$$

$$\vec{f} = \vec{PC}$$

$$L(s) = P + s \times \hat{\vec{d}}_i, s \in [0, 1], i \in \{i \in \mathbb{Z} | 1 \leq i \leq 16\}$$

Discretise time with step $\Delta t = 0.01$ s; at each tick the LOS discriminant $|Q_i - C|^2 - R^2 \geq 0$ is evaluated for all 16 points, and the target is judged hidden only when every inequality is non-negative; summing the successive Δt gives the total screened interval: $\Delta t = 0.05$ s

4.2. Determination of objective function

Maximise $T_{cover_1} = \sum \Delta t \bullet \square \left\{ \forall i, |Q_i(t) - C(t)|^2 - R^2 \geq 0 \right\}$, the cumulative time over which all 16 target points are simultaneously inside the smoke sphere.:

$$T_{cover_1} = \operatorname{argmax} \| t_{exit_1} - t_{enter_1} \|$$

$$T_{\text{cover1}} = t_{1\text{exit}} - t_{1\text{enter}}$$

4.3. Model solution

Code gives shell (17 620, 0, 1 800) and smoke sphere (17 188, 0, 1 736.5); effective screening time is the interval from the first to the last instant all 16 rim points are inside that sphere.

4.4. Results of search

It is found that under the condition given in problem 1, the actual effective shielding time is 1.381s.

5. Problem 2. establishment and solution of the model

5.1. Model building

A two-stage search first brackets (θ, vF_1) by fast bisection, then lets a GA evolve the same pair to maximise screen time, slashing runtime by shrinking the initial space.

5.2. Determination of objective function

Maximize the missile's effective screening time by jointly optimizing the four decision variables—UAV heading, speed, launch instant and detonation instant—in a single objective function:

$$T_{\text{cover}_2} = \operatorname{argmax} \left| \sum_{j \in \left[\frac{t_{\text{det1}}}{\Delta t}, \frac{t_{\text{max}}}{\Delta t} \right]} \mathbb{I} \left(\bigwedge_{i=1}^{16} \Delta_i = 4 \left(\vec{f} \times \hat{d}_i \right)^2 - 4 \left(\|\vec{f}\|^2 - r^2 \right) \geq 0 \right) t \right|_{\Delta}$$

$$t_{\text{max}} = \min \left(\max \left(t_{\text{det1},0,0} + 20, t_{\text{endM}_1} \right) \right)$$

5.3. Model solving

This part will omit the same steps as problem 1 and focus on the application of genetic algorithm based on binary optimization parameter range in problem 2.

Physics restricts heading to 170–180°, speed to 70–140 m s⁻¹, release to 0–2 s and detonation to 1–7 s after release.

Binary search shrinks the angle interval: coarse steps locate the neighbourhood that gives the longest screen time, then the step is halved and the scan repeated; for any parameter x , the next window is centred on the previous best x^* with half the former width:

$$x \in \left[x^* - \frac{\Delta x}{2}, x^* + \frac{\Delta x}{2} \right]$$

Iterative bisection delivered the final tight brackets: heading 177–179°, speed 70–75 m s⁻¹, release ≈ 0.5 s, detonation ≈ 3.3 s; these bounds shrink the GA search space and cut its workload.

The genes are one real-number chromosome: $\theta, v_F, t_{drop}, t_{det}$.

$$\mathbf{x} = [x_1, x_2, x_3, x_4] = [\theta, v_F, t_{drop}, t_{det}]$$

Fitness equals the smoke-screen concealment time returned by the simulator: longer coverage $\Rightarrow$ higher $F(\mathbf{x})$.

Selection probability is proportional to fitness: $p_i = F(x_i) / \sum_j F(x_j)$.

$$p_i = \frac{F(x_i)}{\sum_{j=1}^N F(x_j)}$$

$$X_{ia} = \alpha X_{iA} + (1 - \alpha) X_{iB} \quad X_{ib} = (1 - \alpha) X_{iA} + \alpha X_{iB} \quad \alpha \in [0, 1]$$

$$\mathbf{x}_{best-ever} = \max(\mathbf{x}_{best}, \mathbf{x}_{best-ever})$$

Linear selection favours elite genes, arithmetic crossover blends them, and mutation preserves diversity to escape local optima.

The fittest individual is cloned unchanged into the next generation, and mutated offspring fill the remaining slots.

5.4. Results of search

Drone flight Angle 177.0°, drone flight speed 72.9m/s detonation time 2.5s maximum cover time 4.540s

6. Analysis and testing of the model

6.1. The sensitivity of parameters related to differential evolution is analyzed

Sensitivity analysis of population size: For differential evolution algorithm, population size N affects the exploration ability and computational cost of the algorithm. According to empirical formula:

$$N_0 = \lceil 10 + 2\sqrt{D} \rceil$$

Sensitivity analysis of variation strategy: There are many variation strategies in differential evolution algorithm, and variation has a significant impact on iteration.

6.2. Finally, the sensitivity analysis of the parameters in particle swarm optimization algorithm is carried out

Sensitivity analysis of population size: The relationship between particle swarm optimization algorithm and population size N can be expressed by the following formula:

$$\frac{dP}{dN} = \frac{a}{N} - \frac{b}{N^2}$$

At this time, the critical value of population size N is calculated according to the formula. When the actual N is too small: the diversity is insufficient and the population tends to mature early and converge. When the actual N is too large: the calculation cost increases significantly, but the performance improvement is limited.

6.3. Sensitivity analysis of inertial weights

Exploration of inertial weight control algorithm-development balance, its update formula is:

$$v_i^{t+1} = \omega \cdot v_i^t + c_1 r_1 (p_i^t - x_i^t) + c_2 r_2 (g^t - x_i^t)$$

Mathematical expression of adaptive weight strategy:

$$\omega^t = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \cdot \frac{t}{T}$$

The effects of learning factors can be analyzed by components in the velocity update formula:

$$v_i^{t+1} = \omega v_i^t + \underbrace{c_1 r_1 (p_i^t - x_i^t)}_{\text{Cognitive dimensions}} + \underbrace{c_2 r_2 (g^t - x_i^t)}_{\text{Social divisions}}$$

When c_1 dominates ($c_1 > c_2$): Diversity remains well maintained, but convergence is slow. When c_2 dominates ($c_1 < c_2$): Convergence is fast, but premature.

7. Conclusion

The model's strengths include: 1) Employing a genetic algorithm with differential evolution for Problem 2, where fitness analysis prevents local optimization while differential mutation enhances exploration capabilities. 2) Limitations: Genetic algorithms demonstrate high complexity under multi-parameter conditions, resulting in low time efficiency even after optimization. 3) The greedy strategy improves computational efficiency but relies on empirical rules whose selectivity lacks mathematical validation. 4) The greedy approach cannot mathematically guarantee optimal or near-optimal solutions. Evaluation focuses on solution accuracy and efficiency, providing clear directions for model refinement. The model's advantages lie in its differential evolution-based genetic algorithm, which prevents local optimization through fitness analysis and enhances exploration via differential mutation. Limitations: Genetic algorithms remain computationally intensive under complex multi-parameter conditions, with optimized solutions still showing low time efficiency. The greedy strategy improves computational efficiency but lacks mathematical validation for its selectivity. The greedy approach cannot mathematically guarantee optimal or near-optimal solutions.

Evaluation focuses on solution accuracy and efficiency, providing clear directions for model refinement.

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