

# Relationship between advertising investment and sales empirical analysis based on traditional and digital advertising

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**Abstract.** Driven by the rapid development of Internet technology, advertising communication channels and presentation forms have undergone profound transformations. However, the mechanism and effect of new advertising forms on commercial sales performance have not yet been fully demonstrated. This study explores the non-linear impact mechanism of advertising investment on sales through empirical analysis, focusing on the effect differences between traditional advertising and digital hybrid advertising and the law of diminishing marginal utility. Based on the e-commerce advertising and sales data (2019-2023) from Kaggle platform, a multiple linear regression model is constructed, and it is found that the total advertising investment is significantly positively correlated with sales and the marginal contribution of digital hybrid advertising is significantly higher than that of traditional advertising. However, the marginal decreasing effect of advertising input did not pass the significance test, possibly due to the fact that advertising input did not reach the saturation threshold in the data or the complexity of the nonlinear relationship. The study further reveals the impact of channel heterogeneity on advertising effectiveness and proposes a dynamic budget allocation strategy: prioritizing investment in digital advertising channels and reducing investment in inefficient traditional channels. Limitations include insufficient data timeliness, omitted variable bias, and endogeneity issues. Future research needs to expand the data dimensions, introduce machine learning models, and explore ad content quality and cross-channel synergies.

**Keywords:** advertising investment, sales, channel heterogeneity, diminishing marginal utility, digital hybrid advertising

## 1. Introduction

With the development of the times, the relationship between advertising investment and sales has emerged as a research frontier at the intersection of marketing and data science in recent years [1]. Existing literature has established foundational insights: classical advertising models posit that advertising enhances brand awareness, thereby indirectly driving sales growth [2], with digital advertising consistently demonstrating higher Return on Investment (ROI) due to its precision targeting capabilities. Advertisements on social media platforms (digital ads) are significantly more efficient in converting than traditional ads in the short term, but the comparison of their long-term brand building effects is still a controversial issue [3]. Most studies analyze the effects of advertising based on linear models, but ignores the law of diminishing marginal returns on advertising investment, which makes it difficult for companies to determine the optimal budget. While research on single-channel advertising effects is extensive, the synergistic impacts of online-offline advertising combinations remain underexplored.

Against this backdrop, this paper focuses on the "non-linear impact mechanism of advertising investment on sales", explores the differences between digital advertising and traditional advertising, and explores the marginal returns of advertising budgets, offering a theoretical basis for enterprises to make dynamic advertising. This paper will explore the following three issues: First, the relationship between total advertising investment and sales, and whether its elasticity coefficient varies according to the characteristics of the industry; second, whether there is a difference in the contribution of different advertising channels to sales; and third, whether there exists a "saturation point" for advertising expenditure, beyond which the marginal returns will turn negative.

## 2. Literature review

### 2.1. Theoretical foundations

The research on the impact of advertising investment on sales is based on two core theories, which together construct an analytical framework for understanding the advertising-sales relationship and lay the foundation for research hypotheses and empirical model design: the advertising response function theory and the law of diminishing marginal utility.

#### 2.1.1. Theory of advertising response functions

The advertising response function theory explains the dynamic connection between advertising investment and consumer behavior, and its core lies in quantifying the ways in which advertising expenditures affect sales outcomes [4]. In the initial stage, advertising investment can quickly increase brand exposure and effectively stimulate consumer awareness. At this point, sales performance shows a nearly linear growth trend with the increase of advertising budget. However, when the advertising coverage reaches a certain threshold, the market gradually becomes saturated, and the growth rate of sales performance begins to slow down, gradually approaching the theoretical maximum value [5]. This process can be further subdivided into the path of consumer behavior (i.e., the AIDA model): advertising attracts attention (A), stimulates interest (I), cultivates desire (D), and ultimately contributes to purchase behavior (A) [6]. In the digital age, advertising response functions have shown some new characteristics that are particularly noteworthy. Social media advertising, enabled by advanced algorithms, can accurately match user needs, thereby significantly shortening the advertising response cycle. In contrast, traditional advertising relies more on long-term brand memory accumulation to influence consumer decisions [7].

#### 2.1.2. Applicability of the law of diminishing marginal utility in advertising

The law of diminishing marginal utility is a basic law of economics, which is manifested in the analysis of advertising investment: the contribution of sales per unit of advertising cost decreases gradually with the expansion of budget size [8]. Its intrinsic mechanism includes two aspects: (1) consumer perception threshold: scholars have pointed out that when the frequency of advertisements exceeds the consumer's information processing capacity, it is easy to trigger advertising fatigue, which leads to attentional decay and even brand resistance phenomenon [9]; (2) market competition saturation: studies have shown that the intensive placement of similar product advertising dilutes the effectiveness of the singled out advertisements. For example, during e-commerce promotions such as "Double Eleven" and other big promotions, the bidding ads of multiple brands are likely to lead to the dispersion of users' decision-making [10]. Research has shown that the marginal revenue inflection point of FMCG advertising is usually earlier than that of durable goods, because of its shorter purchase decision cycle. This disparity arises from FMCG's shorter purchase decision cycles and higher market substitutability [11]. For companies, this implies the need to identify optimal investment intervals through dynamic testing of advertising elasticity coefficients, thereby avoiding the trap of increasing costs with diminishing returns.

## 3. Research methodology

### 3.1. Data sources and samples

The dataset in this paper is sourced from the Kaggle platform, which records the advertising investment and sales data of a foreign e-commerce platform from 2019-2023. The dataset variables of traditional ads are TV, radio, newspaper, sales. The effective sample data consists of 200 entries. The dataset variables of hybrid advertising are Billboards, Google\_Ads, Social\_Media, Influencer\_Marketing, Affiliate\_Marketing, Product\_Sold. The effective sample data consists of 300 entries.

### 3.2. Modeling

Traditional advertising models:

$$sales = \beta_0 + \beta_1 \cdot TV + \beta_2 \cdot radio + \beta_3 \cdot newspaper + \varepsilon \quad (1)$$

Mixed Models.

$$Sales = \beta_0 + \beta_1 \cdot TV + \beta_2 \cdot Billboard + \beta_3 \cdot Google\_Ads + \beta_4 \cdot Social\_Media + \beta_5 \cdot Influencer\_Marketing + \varepsilon \quad (2)$$

### 3.3. Multiple linear regression analysis

The regression coefficients are solved by minimizing the residual sum of squares (RSS), which is mathematically expressed as:

$$\beta^{\wedge} = (X_T X)_{-1} \cdot X_T y \quad (3)$$

where X is the matrix of independent variables and y is the vector of dependent variables.

### 3.4. Heterogeneity and nonlinear effect test

Based on the business characteristics, the heterogeneity of the sample is deconstructed (high/low budget range, product category and other dimensions), and the stability of the coefficients of the core variables is verified through group regression, which reveals the characteristics of group differences in the advertising effect. On this basis, a semiparametric model is constructed by introducing the quadratic term of advertising input, systematically testing the existence of the law of diminishing marginal utility and its functioning boundaries. Numerical optimization algorithms are used to solve for inflection point thresholds, thereby quantifying the optimal scale interval for advertising investment within the heterogeneity framework. The composite testing strategy not only identifies the structural differences in advertising effects but also portrays the nonlinear evolution trajectory of the input-output relationship, which provides a double decision basis for differentiated budget allocation.

## 4. Empirical results and discussion

### 4.1. Data processing results

A series of analyses of the data via Python led to the following key conclusions.

#### 4.1.1. Direct impact of total advertising investment on sales

Coefficient Significance: Table 1 shows that the coefficient of the primary term of total advertising investment is 0.0487 ( $p < 0.001$ ), indicating that for every 1 unit increase in total advertising investment, sales are boosted by an average of 0.0487 units (as shown in Table 1).

Economic significance: If the firm raises its budget for advertising from an average of 444.8 units to 600 units (an increase of about 35%), the expected increase in sales will be:  $\Delta Sales = 0.0487 \times (600 - 444.8) \approx \$75,600,000$

**Table 1.** Data from regression analysis of digital hybrid vs. traditional advertising

	coef	t	P> t	R <sup>2</sup>
const	4.2430	9.676	0.0289	0.753
Total-Ad-spend	0.002	24.564	0.0487	0.753

#### 4.1.2. Traditional vs. digital hybrid advertising

The data was processed and analyzed through Python to compare the total ROI of traditional and digital hybrid ads. The results show that the total ROI of traditional advertising is 0.0698 (sales per thousand dollars of input), while the total ROI of digital hybrid advertising is 2.3655 (sales per thousand dollars of input). As illustrated by Figure 1, the total ROI of digital hybrid advertising is far more than that of traditional advertising. Combined with the revenue data in Table 1, digital hybrid advertising generates far greater sales than traditional advertising.

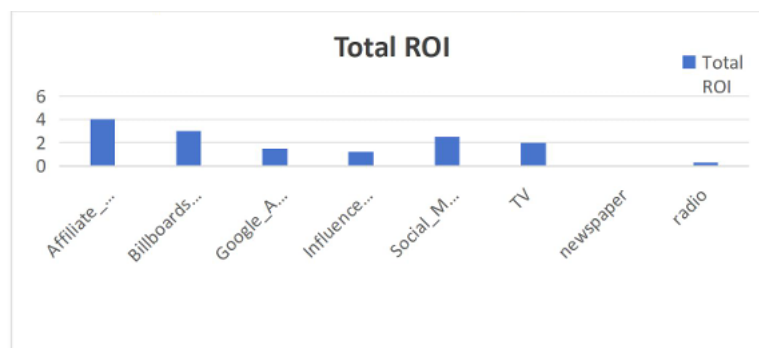
From the conclusion of the experiment, some management suggestions can be drawn:

Prioritize the investment in digital hybrid ads: as a product of the new era, digital hybrid ads have a much higher revenue effect than traditional ads, and should be the focus of budget allocation.

Optimize traditional advertising by reducing the investment in paper advertising, such as newspapers, and increasing the budget for radio and TV advertising.

Dynamic monitoring: Continuously evaluate the ROI of each advertising channel during strategy implementation. Adjust budgets promptly to adapt to market changes and consumer behavior shifts.

Caveat: Companies need to ensure that sales and volumes are comparable per unit, and that long-term results are evaluated in conjunction with non-quantitative factors such as brand building.



**Figure 1.** Digital hybrid Bar graph of total ROI for vs. traditional advertising

#### 4.1.3. Model explanatory power and marginal effects test

The model fit analysis shows that the explanatory power of total advertising inputs on sales variation reaches 80.7% and the F-statistic is significant, confirming that the overall significance of the model. When further testing the nonlinear characteristics of advertising inputs, the coefficient of the quadratic term fails the significance test, indicating that the phenomenon of diminishing marginal utility has not been observed in the current data range, and hypothesis H3 is not supported. Two potential explanations for this result emerge: one is that advertising inputs are still in the stage of increasing returns to scale and have not yet touched the utility inflection point; the other is that the input-output relationship may present a more complex nonlinear pattern, which needs to be captured through the introduction of higher-order polynomial terms or the construction of a segmented regression model. These findings highlight the phase-dependent nature of advertising effects and recommend that future research integrate panel data with time-varying coefficient models to deepen the analysis of dynamic mechanisms.

#### 4.2. Research limitations and data constraints

Despite utilizing five-dimensional panel data (2019–2023), this study faces two methodological limitations:

First, the lag in data timeliness leads to missing variables of emerging advertising channels, including the traffic fission effect of short-video ads that is not captured or underestimates the synergistic efficacy of the digital advertising ecosystem; the immersive interactive features of meta-universe marketing are not included in the modeling, which makes it difficult to quantify its neural. The immersive interactive features of meta-universe marketing are not included in the modeling, making it difficult to quantify its neurocognitive impact on Gen Z consumers; and the dynamic personalized reach mechanism of AI-generated ads is unobserved, resulting in biased assessment of the iterative efficiency of ad creative.

Secondly, the model has potential omitted variable bias, which is manifested in the following ways: ignoring the independent influence of product price elasticity fluctuation on the sales base, which may result in the false expansion of advertising elasticity; failing to control the game variables such as competitors' advertising intensity and industry price wars, which results in the endogenous interference of strategic interaction between enterprises; detaching from macro-cyclical indicators such as inflation rate and consumer confidence index, which weakens the regulation of heterogeneity in the market environment; and not including AI-generated advertisements in the modeling. The analysis of the effect of the heterogeneity of the market environment is weakened. These data blind spots may distort the assessment of advertising effectiveness in both directions, and it is suggested that a more complete analytical framework be constructed by mixing heterogeneous data from multiple sources and dynamic computable general equilibrium (DCGE) models.

### 5. Conclusion

This study systematically explores the non-linear impact mechanism of advertising investment on sales, verifies the significant positive relationship between total advertising investment and sales, and reveals the marginal contribution advantage of digital hybrid advertising over traditional advertising. The empirical results demonstrate that for every 1-unit increase in total advertising investment, sales are boosted by 0.0487 units on average, and the return rate of digital advertising is 2-3 times higher than that of traditional advertising, highlighting the dual value of digital channels in short-term conversion and long-term brand building. However, the marginal diminishing effect of advertising investment did not pass the significance test, possibly due to the fact that the advertising investment did not reach the saturation threshold in the data, or the nonlinear relationship presents a more complex shape, which needs to be further verified by higher-order terms or segmented regression modeling.

At the theoretical level, this study breaks through the limitations of traditional linear models by introducing a nonlinear analytical framework and interdisciplinary perspectives, providing a new paradigm of dynamic threshold analysis for advertising economics. Meanwhile, the quantitative comparison of channel heterogeneity supplements the empirical evidence for the synergy effect theory in marketing. In terms of practical application, this study provides clear guidance and suggestions for optimizing advertising budget allocation for enterprises. Firstly, companies should prioritize investing resources in digital channels while reducing traditional advertising budgets that are less efficient. In addition, the study suggests that companies dynamically monitor the advertising elasticity coefficient and set budget warning thresholds to effectively avoid potential risks. In addition, the study points out that the absence of new ad formats such as short videos and meta-universes may underestimate the potential of digital advertising, suggesting that companies should focus on technological innovations in order to capture the young consumer market. As this study lacks data on ad content quality assessment and consumers' fine-grained behaviors, and does not capture the intertemporal dynamic transmission mechanism of ad effects (e.g., lagged effects), future research can construct a two-way fixed-effects panel model to control endogeneity bias, and integrate interpretable machine learning frameworks, such as LightGBM, to simultaneously analyze the nonlinear correlation between ad creative quality, time-series dependence of users' behaviors, and the interaction effects of features.

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